NEWCASTLE UNIVERSITY

SEMESTER 2 2014/2015

TIME SERIES ECONOMETRICS

Time allowed - 90 minutes

Answer THREE out of the four questions.

Each question is worth 1/3 of total marks.

Question 1

Consider the random variable Y with probability mass function

$$\Pr(Y = y) = \frac{\exp(-\lambda)\lambda^y}{y!}, \ \lambda > 0, \ y = 0, 1, 2, \ldots$$

You remember that $\mathrm{E}(Y) = \mathrm{V}(Y) = \lambda$. Let $y_1, y_2, \ldots, y_n$ be a random sample from this distribution.

a) What is the name of this distribution? For what type of applications can it be used? Give an example of a possible application.

b) Derive the likelihood function and the log-likelihood function.

c) Derive expressions for the score, the Hessian and the information.

d) Show that the expectation of the score is zero.

e) Derive the ML estimator for λ.

f) Derive the Cramer-Rao lower bound of the ML estimator.

g) Does the ML estimator reach the Cramer-Rao lower bound?

h) Assume $n = 20$ and $\sum_{i=1}^{20} y_i = 20$. Test whether $\lambda = 2$ using both the Wald and Score tests at a 5% significance level.

Question 2

a) Define the terms autocorrelation function (acf) and partial autocorrelation function (pacf).

b) What is the difference between an autocorrelation function (acf) and a sample autocorrelation function (sample acf)?

c) You obtain the following sample autocorrelations and sample partical autocorrelations for a series of 225 observations:

Lag	1	2	3	4	5	6	7	8
sample acf	0.61	0.61	0.47	0.45	0.39	0.37	0.32	0.32
sample pacf	0.61	0.38	0.00	0.15	0.05	0.04	0.04	0.05

i) Which of these sample autocorrelations and sample partial autocorrelations are significant at a 5% significance level?

ii) Given your result from i), what would be a good time series process for this data?

iii) Check whether the first two sample autocorrelation coefficients are jointly significantly different from zero using the Ljung-Box test at a 5 % significance level.

d) You have estimated the following ARMA(1,2) model:

$$y_t = \frac{1}{2}y_{t-1} + \varepsilon_t + \frac{1}{8}\varepsilon_{t-1} + \frac{1}{2}\varepsilon_{t-2}, \quad \varepsilon_t \overset{iid}{\sim} \mathrm{N}(0, \sigma^2).$$

i) Is this model stationary?

ii) Derive the 1-step-ahead and 2-step-ahead forecasts.

iii) Derive the 1-step-ahead and 2-step-ahead forecast errors.

iv) Are the forecast errors from iii) corellated?

e) Consider the AR(1) process

$$y_t = \beta y_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} \mathrm{N}(0, \sigma^2)$$

with $-1 < \beta < 1$. Let $w_t = \Delta y_t = y_t - y_{t-1}$. Derive $\mathrm{V}(w_t)$.

[Hint: $\mathrm{V}(A - B) = \mathrm{V}(A) + \mathrm{V}(B) - 2\,\mathrm{Cov}(A, B)$]

Question 3

Consider the GARCH(1,1) model:

$$\varepsilon_t = \nu_t \sqrt{h_t}, \quad \nu_t \overset{iid}{\sim} \mathrm{N}(0,1),$$
$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1},$$

with $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$.

a) Derive the conditional mean $\mathrm{E}_{t-1}(\varepsilon_t)$ and the conditional variance $\mathrm{V}_{t-1}(\varepsilon_t^2)$.

b) Derive the (unconditional) mean $\mathrm{E}(\varepsilon_t)$ and the (unconditional) variance $\mathrm{V}(\varepsilon_t^2)$.

c) Define $\eta_t = \varepsilon_t^2 - h_t$ and remember that η_t is an uncorrelated series with zero mean (you do not need to prove this). Which process does the $\{\varepsilon_t^2\}$ sequence follow?

d) Derive the 1-step-ahead and 2-step-ahead forecasts of the conditional variance.

Next you fit a GARCH(1,1) model to daily logarithmic returns of the FTSE 100 index. Here is part of the output:

```
Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(1,1)
Mean Model : ARFIMA(0,0,0)
Distribution : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
omega   0.000004    0.000001   7.5075        0
alpha1  0.065918    0.008479   7.7744        0
beta1   0.863779    0.018348  47.0778        0

LogLikelihood : 879.0964

Information Criteria
------------------------------------
Akaike       -6.9257
Bayes        -6.8838

Q-Statistics on Standardized Residuals
------------------------------------
              statistic p-value
Lag[1]           0.1236  0.7252
Lag[p+q+1][1]    0.1236  0.7252
Lag[p+q+5][5]    6.1507  0.2918
d.o.f=0
H0 : No serial correlation
```

```
Q-Statistics on Standardized Squared Residuals
------------------------------------
               statistic p-value
Lag[1]            0.3071  0.5795
Lag[p+q+1][3]     1.5886  0.2075
Lag[p+q+5][7]     2.7330  0.7411
d.o.f=2

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.9853   2  0.6110
ARCH Lag[5]     2.5346   5  0.7713
ARCH Lag[10]    9.6939  10  0.4677

Sign Bias Test
------------------------------------
                   t-value    prob sig
Sign Bias           1.8504 0.06545   *
Negative Sign Bias  0.7197 0.47238
Positive Sign Bias  0.1367 0.89137
Joint Effect        5.0584 0.16757
```

e) Assess whether the GARCH(1,1) model adequately fits the data.

Question 4

Discuss in detail univariate GARCH models. Make sure you address the following points: estimation, model selection, model evaluation and model extensions.

Critical Values of the Chi-Square Distribution with ν Degrees of Freedom

	Significance Level		
ν	0.10	0.05	0.01
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.42	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89

END OF PAPER